

Baryonic Bound State of Vortices in Multicomponent Superconductors

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Abstract

We construct a bound state of three $1/3$ -quantized Josephson coupled vortices in three-component superconductors with intrinsic Josephson couplings, which may be relevant with regard to iron-based superconductors. We find a Y-shaped junction of three domain walls connecting the three vortices, resembling the baryonic bound state of three quarks in QCD. The appearance of the Y-junction (but not a Δ -junction) implies that in both cases of superconductors and QCD, the bound state is described by a genuine three-body interaction (but not by the sum of two-body interactions). We also discuss a confinement/deconfinement phase transition.

Keywords: multi-band superconductor, vortices, fractional flux quanta, Ginzburg-Landau free energy, interband phase difference soliton, confinement, baryon, meson, QCD.

1 Introduction

Color confinement in quantum chromodynamics (QCD) is one of the most challenging problems in modern physics. Quarks having fractional electric charges $\pm(1/3)e$ or $\pm(2/3)e$ should be confined by color electric fluxes to form hadrons (mesons or baryons) with integer electric charges. As a result, only mesons and baryons, made of two and three quarks, respectively, can be observed. A color electric flux tube stretched between a quark and an anti-quark provides constant attractive force or a potential that is linearly dependent on the distance between the quarks. A recent lattice QCD simulation has confirmed this picture [1]. On the other hand, one can imagine two possible configurations of color fluxes in a baryon: a Y-shaped junction or a Δ -shaped junction connecting three quarks. The Δ -junction implies that the three-quark interaction is described by the sum of two-body interactions while the Y-junction implies a genuine three-body interaction. Although it has been a long standing issue as to which of these possible configurations is the actual one, a further study of lattice QCD simulations has clearly demonstrated the Y-junction [2].

A bound state of Josephson coupled vortices confined by domain walls exists in multicomponent superconductors, and it resembles the bound states of quarks confined by fluxes. Half-quantized vortices stably exist in two-band superconductors when the interband Josephson coupling is negligible [3]. When the interband Josephson coupling is taken into account, a half-quantized vortex is attached by a domain wall [4, 5] which extends to the edge of the sample. Since the domain wall pulls the vortex in order to reduce the energy, the half-quantized vortex is unstable. On the other hand, two half-quantized vortices winding around two different gap functions are connected by a domain wall (sine-Gordon kink) [4, 5] when the interband Josephson coupling is considered. The domain wall provides an attractive potential linearly depending on the distance between the two vortices [6]; this bound state resembles a meson in QCD. In fact, confinement/deconfinement phase transition occurs at finite temperature, similar to the case in QCD, in which fluctuations of the domain wall contribute to entropy [6]. That is, vortices with fractional quanta are confined to become vortices with integer quanta below a certain temperature, while they are deconfined above that temperature. However, the question is: Is there any model or actual material that indicates a baryonic bound state of vortices? In the absence of intrinsic Josephson terms, vortex bound states have been discussed in multi-component superconductors [7]. How are they connected by domain walls, once the Josephson terms are turned on?

In this paper, we show that baryonic bound states indeed exist in three-component superconductors with intrinsic Josephson terms. We show that three $1/3$ -quantized vortices are connected by a Y-shaped junction of domain walls, resembling a baryon in QCD. We also discuss a confinement/deconfinement phase transition. Above the certain critical temperature, the three fractional vortices are deconfined from a vortex baryon.

Some two-band superconductors exhibit type 1.5 superconductivity, i.e., repulsion at a short distance and attraction at a large distance between two integer vortices [8]. Such a structure leads to a cluster of vortices, which has been experimentally confirmed in MgB_2 [9, 10]. Recently, three-component superconductors have attracted considerable attention because of the discovery of iron-based superconductors [11]. Therefore, we propose the multi-band superconductors to test baryonic bound states of vortices with $1/3$ quanta which exhibit a confinement/deconfinement transition. Our solution will also suggest the possibility of experimentally determining intrinsic

Josephson couplings by determining the shape (angles and lengths) of a Y-junction of three vortices.

2 Three-component superconductors

Multicomponent (n -component) superconductors can be generically described by the Ginzburg-Landau free energy,

$$F = \sum_{i=1}^n \left[\frac{\hbar^2}{4m} \left| \left(\nabla + i \frac{2e}{\hbar c} \mathbf{A} \right) \Psi_i \right|^2 + \frac{\lambda_i}{4} (|\Psi_i|^2 - v_i^2)^2 \right] + F_J + \frac{\mathbf{H}^2}{8\pi}, \quad (1)$$

with the intrinsic Josephson terms

$$F_J = - \sum_{i \neq j} \frac{1}{2} \gamma_{ij} (\Psi_i^* \Psi_j + \Psi_j^* \Psi_i) = - \sum_{i \neq j} \gamma_{ij} |\Psi_i| |\Psi_j| \cos(\theta_i - \theta_j). \quad (2)$$

Here, γ_{ij} are constants and Ψ_i is decomposed into amplitude and phase as $\Psi_i = |\Psi_i| e^{i\theta_i}$. The phases of Ψ_i and Ψ_j preferably coincide for $\gamma_{ij} > 0$ while they tend to have π phase difference for $\gamma_{ij} < 0$. All phases are the same in the ground state when $\gamma_{ij} > 0$ for all i and j , while the system is frustrated when $\gamma_{ij} < 0$ for all i and j . The Hamiltonian (1) is invariant under gauge symmetry,

$$\mathbf{A} \rightarrow \mathbf{A} - \frac{\hbar c}{2e} \nabla \theta(\mathbf{x}), \quad \Psi_i \rightarrow e^{i\theta(\mathbf{x})} \Psi_i. \quad (3)$$

In the limit of $\gamma_{ij} = 0$, the Hamiltonian (1) enjoys $U(1)^n$ symmetry, of which the overall phase rotation exhibits the gauge symmetry (3) while others exhibit global symmetry. In this case, there appear $n - 1$ Nambu-Goldstone modes. They are gapped for non-zero γ_{ij} (the Legget modes).

Hereafter, we restrict ourselves to three-component ($n = 3$) superconductors in two space dimensions [12, 13]. We consider the positive Josephson couplings, $\gamma_{ij} > 0$, while the negative Josephson couplings, $\gamma_{ij} < 0$, give frustrated systems [13]. There exist three types of vortices, labeled as $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, winding around the first, second, and third component by 2π , respectively. The energy of each vortex is logarithmically divergent when $\gamma_{ij} = 0$ and linearly divergent when $\gamma_{ij} \neq 0$, if the system size is infinite. Let us consider that the $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ vortices are placed at the edges (P_1 , P_2 , and P_3 , respectively) of a triangle, as shown in Fig. 1. In the large circle, the total configuration is the integer vortex $(1, 1, 1)$, which implies that the total energy is finite. In other words, the integer vortex has an internal structure made of three vortices, all of which are $1/3$ quantized, as shown below.

Instead of the $U(1)^3$ generators $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, let us prepare four linearly dependent generators: the gauge rotation $(1, 1, 1)$ and three gauge-invariant rotations $(0, -1, 1)$, $(1, 0, -1)$, and $(-1, 1, 0)$. Among these, only the gauge rotation is accompanied by gauge transformation (3), while the others are all global phase rotations. In these new generators, the

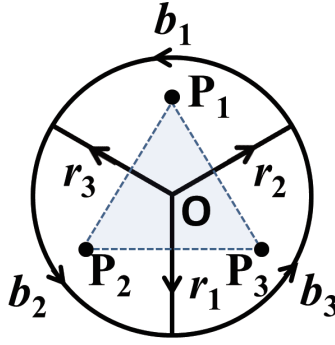


Figure 1: (Color online) The $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ vortices are placed at P_1 , P_2 and P_3 , respectively. b_i ($i = 1, 2, 3$) corresponds to $1/3$ circles at the boundary, and r_i corresponds to the radial paths from the origin O to the circle at the boundary. The $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ vortices are encircled by $b_1 - r_3 + r_2$, $b_2 - r_1 + r_3$, and $b_3 - r_2 + r_1$, respectively.

winding of the $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ vortices can be decomposed into

$$\begin{aligned}
 P_1 : \quad (1, 0, 0) &= \frac{1}{3}(1, 1, 1) + 0(0, -1, 1) + \frac{1}{3}(1, 0, -1) - \frac{1}{3}(-1, 1, 0), \\
 P_2 : \quad (0, 1, 0) &= \frac{1}{3}(1, 1, 1) - \frac{1}{3}(0, -1, 1) + 0(1, 0, -1) + \frac{1}{3}(-1, 1, 0), \\
 P_3 : \quad (0, 0, 1) &= \frac{1}{3}(1, 1, 1) + \frac{1}{3}(0, -1, 1) - \frac{1}{3}(1, 0, -1) + 0(-1, 1, 0).
 \end{aligned} \tag{4}$$

We see that all the paths b_i ($i = 1, 2, 3$) in Fig. 1 correspond to $2\pi/3$ rotation of the gauge generator $(1, 1, 1)$ with Eq. (3) and consequently that these vortices are all $1/3$ quantized; their magnetic flux is $\Phi_0/3$ with the unit flux quanta $\Phi_0 = hc/2e$ [14]. The $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ vortices are encircled by $b_1 - r_3 + r_2$, $b_2 - r_1 + r_3$, and $b_3 - r_2 + r_1$, respectively (Fig. 1). Therefore, we can identify the paths $\pm r_1$, $\pm r_2$ and $\pm r_3$ corresponding to $\pm 2\pi/3$ of the global phase rotations $(0, -1, 1)$, $(1, 0, -1)$, and $(-1, 1, 0)$, respectively. The global phase rotation along the radial path r_i can be written up to constant phases as

$$\begin{aligned}
 r_1 : \quad \Psi_1 &= |\Psi_1|, \quad \Psi_2 = e^{-(2\pi i/3)f(r)}|\Psi_2|, \quad \Psi_3 = e^{(2\pi i/3)f(r)}|\Psi_3|, \\
 r_2 : \quad \Psi_1 &= e^{(2\pi i/3)f(r)}|\Psi_1|, \quad \Psi_2 = |\Psi_2|, \quad \Psi_3 = e^{-(2\pi i/3)f(r)}|\Psi_3|, \\
 r_3 : \quad \Psi_1 &= e^{-(2\pi i/3)f(r)}|\Psi_1|, \quad \Psi_2 = e^{(2\pi i/3)f(r)}|\Psi_2|, \quad \Psi_3 = |\Psi_3|,
 \end{aligned} \tag{5}$$

where a function $f(r)$ has the boundary conditions $f(r = 0) = 1$ and $f(r \rightarrow \infty) = 0$.

The integer vortex configuration $(1, 1, 1)$ is of the Abrikosv type which has finite energy (due to the fact that the associated $U(1)$ symmetry is fully local), whereas the fractional vortices correspond to global ungauged symmetries and hence they have a logarithmically divergent energy, even in the absence of the Josephson terms, $\gamma_{ij} = 0$. This is why the fractional vortices are confined whereas the integer vortices are acceptable finite-energy solutions.

In the presence of the Josephson terms, $\gamma_{ij} \neq 0$, we expect there to be a sine-Gordon kink in each path r_i that connects two vortices. However, the question that arises is: how does it connect two vortices? Does it connect along the segment $P_j P_k$?

3 Baryonic bound state

We concentrate on the case of $\gamma_{ij} > 0$ for all i and j , which may be the case of iron-based superconductors. In this case, all phases are the same in the ground state, which is unique with respect to gauge transformation (3). The phases at $r \rightarrow \infty$ are the same, which we consider to be zero according to the gauge symmetry; hence, Eq. (5) holds, including for the constant phases. Along each radial path, the Josephson term in Eq. (2) can be written as $\gamma_{ij}|\Psi_i||\Psi_j|\cos((4\pi/3)f(r))$, from Eq. (5). We thus find that it takes a non-zero value $\frac{1}{2}\gamma_{ij}|\Psi_i||\Psi_j|$ at the center ($r = 0$). Therefore, the linear connection of vortices along the segments P_jP_k due to the sine-Gordon kinks would increase energy because a domain (membrane) with finite energy appears inside the triangle $P_1P_2P_3$. The sine-Gordon kinks should bend to form the Y-junction.

In fact, the numerical solution indicates the Y-junction, as seen in Fig. 2. For simplicity, we have taken $\gamma_{ij} = \gamma (> 0)$ but the general case is straightforward. In Fig. 2-(c), we find that the magnetic field is localized at the center of each vortex. Fig. 3-(a) shows the phases of Ψ_1 , Ψ_2 , and Ψ_3 , indicating a phase winding at P_i . We also obtain the numerical solution for the function $f(r)$ in Eq. (5), (Fig. 3-(b)). In this numerical simulation, we have fixed the positions of the three vortices. The wall tension leads to a linear potential (confining force), and these vortices collapse to form a single integer vortex.

4 Confinement/deconfinement phase transition

Here we discuss that the Y-junction can be stable at finite temperature and exhibits a phase transition, as proposed by Goryo *et.al.* [6] for a mesonic bound state of vortices in two-gap superconductors. To this end, we construct the effective theory for the phases of the gap functions with keeping the amplitudes constants, $\Psi_i(\mathbf{x}) = v_i \exp(i\theta_i(\mathbf{x}))$, given by

$$F_{\text{eff.}} = \left[\sum_i \frac{\hbar^2 v_i^2}{4m} \left(\nabla \theta_i + i \frac{2e}{\hbar c} \mathbf{A} \right)^2 + \sum_{i \neq j} v_i v_j \gamma_{ij} (1 - \cos(\theta_i - \theta_j)) \right], \quad (6)$$

up to a constant. Let us define the phase differences as gauge invariant dynamical variables as

$$\phi_1 \equiv \theta_2 - \theta_3, \quad \phi_2 \equiv \theta_3 - \theta_1, \quad \phi_3 \equiv \theta_1 - \theta_2, \quad (\phi_1 + \phi_2 + \phi_3 = 0). \quad (7)$$

With taking a gauge fixing as

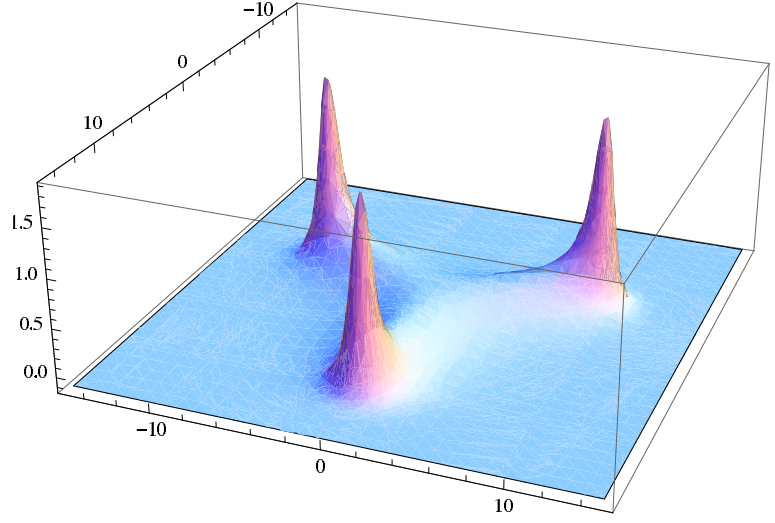
$$\theta_1 + \theta_2 + \theta_3 = 0, \quad (8)$$

the energy (6) is further reduced to

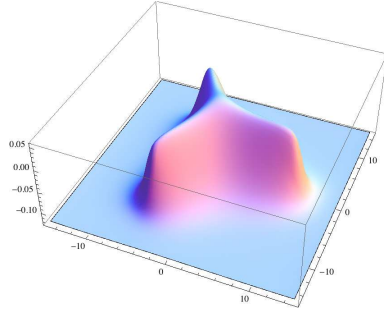
$$F_{\text{eff.}} = \sum_i \left[\frac{\hbar^2 v_i^2}{12m} (\nabla \phi_i)^2 + \eta_i^2 (1 - \cos \phi_i) \right], \quad (9)$$

where we have set $\mathbf{A} = 0$ and defined

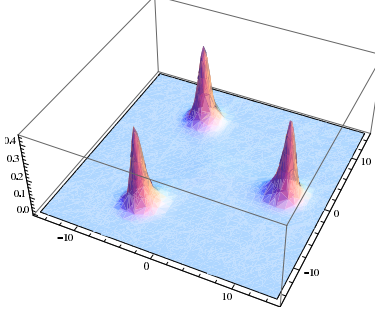
$$\eta_1^2 \equiv v_2 v_3 \gamma_{23}, \quad \eta_2^2 \equiv v_3 v_1 \gamma_{31}, \quad \eta_3^2 \equiv v_1 v_2 \gamma_{12}. \quad (10)$$



(a)



(b)



(c)

Figure 2: (Color online) Baryonic bound state of vortices. Plots of (a) the total energy density, (b) the energy density of the Josephson couplings, and (c) the magnetic field. For simplicity, we take $\gamma_{ij} = \gamma > 0$, but the general case is straightforward. The relaxation method has been used with parameters $\hbar = c = 2m = 2e = v = \lambda/2 = 1$ and $\gamma = 0.02$.

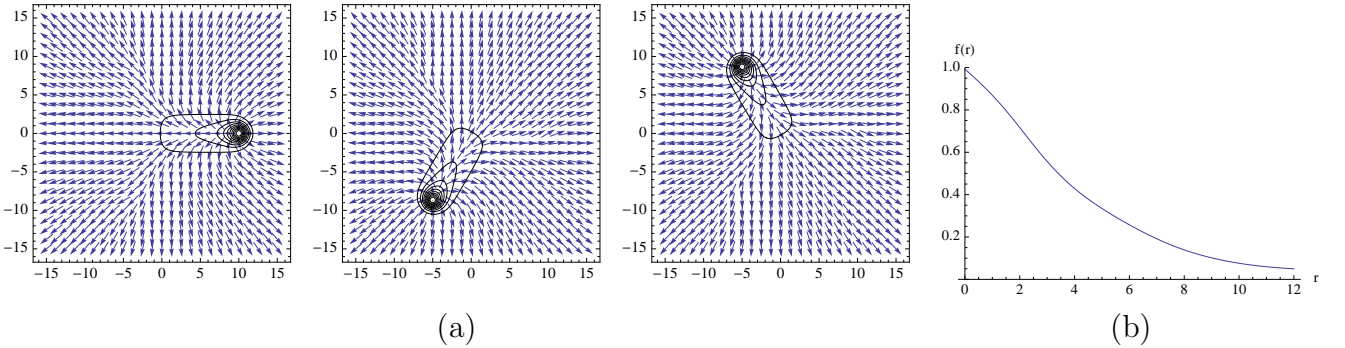


Figure 3: (Color online) (a) The arrows and contour lines indicate, respectively, the phases and amplitudes of Ψ_1 (left), Ψ_2 (middle) and Ψ_3 (right). (b) The plot of the function $f(r)$.

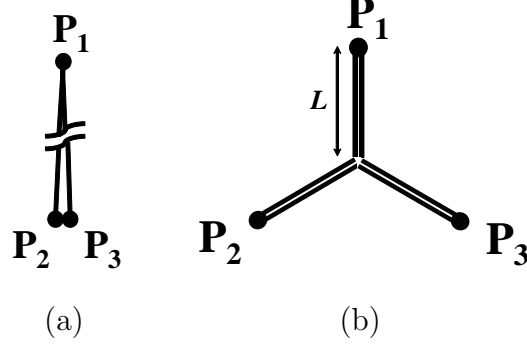


Figure 4: (a) The two vortices P_2 $(0, 1, 0)$ and P_3 $(0, 0, 1)$ together are placed at the same position very far from the vortex P_1 $(1, 0, 0)$. They are connected by a sine-Gordon domain wall. (b) The most symmetric configuration at a finite temperature.

In order to calculate the tension of the domain wall attached to the vortex $(1, 0, 0)$, let us place the two vortices P_2 $(0, 1, 0)$ and P_3 $(0, 0, 1)$ together at the same position which is very far from the vortex P_1 $(1, 0, 0)$ as in Fig. 4(a). In this situation, we can set $\theta_2 = \theta_3$ so that we have

$$\phi_1 = 0, \quad \phi_2 = -\phi_3 \equiv \phi. \quad (11)$$

Then, the effective model (9) reduces to the sine-Gordon model

$$F_{\text{eff}}^{(1)} = K^{(1)}(\nabla\phi)^2 + \frac{\Gamma^{(1)}}{2}(1 - \cos\phi),$$

$$K^{(1)} \equiv \frac{\hbar^2(v_2^2 + v_3^2)}{12m}, \quad \Gamma^{(1)} \equiv 2(\eta_2^2 + \eta_3^2). \quad (12)$$

This can be rewritten as the Bogomol'nyi form

$$\begin{aligned} F_{\text{eff}}^{(1)} &= [K^{(1)}(\nabla\phi)^2 + \Gamma^{(1)}\sin^2(\phi/2)] \\ &= \left(\sqrt{K^{(1)}}\nabla\phi \pm \sqrt{\Gamma^{(1)}}\sin(\phi/2)\right)^2 \mp 2\sqrt{K^{(1)}\Gamma^{(1)}}\nabla\phi\sin(\phi/2) \\ &\geq \varepsilon \end{aligned} \quad (13)$$

with the topological charge density in the second term in the second line,

$$\varepsilon \equiv \mp 2\sqrt{K^{(1)}\Gamma^{(1)}}\nabla\phi\sin(\phi/2) = \pm 4\sqrt{K^{(1)}\Gamma^{(1)}}\nabla\cos(\phi/2). \quad (14)$$

The most stable configurations with the minimum energy can be achieved by satisfying the Bogomol'nyi equation, obtained by $(\dots)^2 = 0$ in the second line of Eq. (13), *i.e.*

$$\sqrt{K^{(1)}}\nabla\phi \pm \sqrt{\Gamma^{(1)}}\sin(\phi/2) = 0. \quad (15)$$

One (anti-)kink solution can be obtained as

$$\phi = 4 \arctan \exp \left[\pm \frac{1}{4} \sqrt{\frac{\Gamma^{(1)}}{K^{(1)}}} (x - x_0) \right], \quad (16)$$

where x is the coordinate perpendicular to the kink and x_0 denotes the position of the kink. The tension of the one (anti-)kink is

$$\begin{aligned} T^{(1)} &= \left| \int dx \varepsilon \right| = 4\sqrt{K^{(1)}\Gamma^{(1)}} \left| [\cos(\phi/2)]_{x=-\infty}^{x=+\infty} \right| = 8\sqrt{K^{(1)}\Gamma^{(1)}} \\ &= 8\sqrt{\frac{\hbar^2(v_2^2 + v_3^2)(\eta_2^2 + \eta_3^2)}{12m}} = 8\hbar\sqrt{\frac{v_1(v_2^2 + v_3^2)(v_2\gamma_{21} + v_3\gamma_{31})}{6m}}. \end{aligned} \quad (17)$$

In the same way, the other two domain walls attached to the $(0, 1, 0)$ and $(0, 0, 1)$ have the tensions

$$\begin{aligned} T^{(2)} &= 8\sqrt{K^{(2)}\Gamma^{(2)}}, \quad K^{(2)} \equiv \frac{\hbar^2(v_3^2 + v_1^2)}{12m}, \quad \Gamma^{(2)} \equiv 2(\eta_3^2 + \eta_1^2), \\ T^{(3)} &= 8\sqrt{K^{(3)}\Gamma^{(3)}}, \quad K^{(3)} \equiv \frac{\hbar^2(v_1^2 + v_2^2)}{12m}, \quad \Gamma^{(3)} \equiv 2(\eta_1^2 + \eta_2^2), \end{aligned} \quad (18)$$

respectively.

We are now ready to discuss the confinement/deconfinement phase transition. For simplicity, we consider the most symmetric case with

$$v_1 = v_2 = v_3 \equiv v, \quad \gamma_{12} = \gamma_{23} = \gamma_{31} \equiv \gamma. \quad (19)$$

The tension of each domain wall becomes

$$T_{\text{dw}} = 8\hbar v^2 \sqrt{\frac{2\gamma}{3m}}. \quad (20)$$

In this case, the molecule is \mathbf{Z}_3 symmetric as in Fig. 4(b). For each domain wall with the length L , the total energy and entropy can be evaluated as [6, 18]

$$E = T_{\text{dw}}L, \quad S_{\text{dw}} = \frac{k_B \ln 2\pi}{\xi}L, \quad (21)$$

respectively, with a short length cut-off ξ which is the largest among the coherence length and the penetration depth. Consequently, the free energy of each domain wall at the temperature T is given by

$$F_{\text{dw}} = E_{\text{dw}} - TS_{\text{dw}} = \left(T_{\text{dw}} - \frac{k_B T \ln 2\pi}{\xi} \right) L = AL, \quad (22)$$

$$A \equiv T_{\text{dw}} - \frac{k_B T \ln 2\pi}{\xi}. \quad (23)$$

When the coefficient A is positive, the interger vortex is stable, *i.e.*, in the confinement phase. On the other hand, when the coefficient A is negative, the integer vortex tends to be split into a set of the three fractional vortices in order to reduce the free energy of the domain walls, that is, the deconfinement occurs. Therefore, the critical temperature for the deconfinement is found to be

$$T_{\text{crit}} = \frac{\xi T_{\text{dw}}}{k_B \ln 2\pi} = \frac{16\xi\hbar v^2}{k_B \ln 2\pi} \sqrt{\frac{2\gamma}{3m}}. \quad (24)$$

This expression is the same with Goryo *et.al* [6]. In the most symmetric case with Eq. (19) which we are considering, the confinement mechanism is essentially the same with the case of two-gap superconductors.

5 Summary and Discussion

In summary, we have constructed baryonic states of three $1/3$ -quantized vortices and have found that these vortices are connected by the Y-junction of domain walls, resembling a baryonic bound state of three quarks in QCD. In both cases of superconductors and QCD, the appearance of the Y-junction and not of a Δ -junction implies that the bound state is described by a genuine three-body interaction and not by the sum of two-body interactions. The confinement/deconfinement transition of vortices has been studied. This common feature between superconductors and QCD will shed a new light on the color confinement problem of QCD.

The similarities between superconductors and QCD should be further clarified. As a toy model of QCD, Shifman and Unsal [19] considered an $SU(2)$ gauge theory in three space dimensions with one direction compactified as $\mathbf{R}^2 \times S^1$. The theory becomes a $U(1)$ gauge theory in two dimensional space \mathbf{R}^2 in the limit of a small radius of S^1 . It was shown by Polyakov [20] that the confinement occurs in a $U(1)$ gauge theory in two dimensional space; electrically charged particles (quarks) are confined by an electric flux. By taking a duality, quarks are mapped into vortices, while electric fluxes are mapped into sine-Gordon domain walls, so that a meson made of two quarks is mapped to a mesonic bound state of two vortices. There, the quark confinement can be understood as the vortex confinement which is described by two-gap superconductors. The confinement is nontrivial in the former, while it can be easily shown in the latter. We therefore expect that a discussion along the same line shows that a baryon made of three quarks in $SU(3)$ QCD is mapped to a bayonic bound state of three vortices found in this paper. We conjecture that the existence of a Y-junction in a baryon in QCD can be shown by using a duality map to three-gap superconductors. In multi-gap superconductors with more than three gaps, the bound states of more vortices should exist which may correspond to tetraquarks, pentaquarks, etc. in QCD.

Although we have chosen the same Josephson couplings $\gamma_{ij} = \gamma$ as an example, an extension to the general case is straightforward. It may be useful to determine intrinsic Josephson couplings of multicomponent superconductors such as iron-based superconductors by determining the shape of the Y-junction. Several interesting studies have been conducted on for vortex mesons in two-component or p -wave superconductors, for instance, the studies on a lattice of vortex mesons and twistons [21] and those on vortex clusters [9, 10]. We hope that our work will stimulate further theoretical and experimental studies of multicomponent superconductors, particularly iron-based ones.

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